

## TEST OF MATHEMATICS FOR UNIVERSITY ADMISSION

### PAPER 1

### SPECIMEN

Time: 75 minutes

Additional Materials: Answer sheet

### INSTRUCTIONS TO CANDIDATES

**Please read these instructions carefully, but do not open the question paper until you are told that you may do so.**

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

This paper is the first of two papers.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Please complete the answer sheet with your candidate number, centre number, date of birth, and full name.

Calculators must **NOT** be used. There is no formulae booklet for this test.

**Please wait to be told you may begin before turning this page**

This question paper consists of 12 printed pages and 4 blank pages

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1. The sum of the two values of  $x$  that satisfy the simultaneous equations

$$x - 3y + 1 = 0 \text{ and } 3x^2 - 7xy = 5 \text{ is}$$

- A -8.5
- B -7.5
- C -1.5
- D 3.5
- E 4.5
- F 5

2. The number of solutions in the interval  $0 \leq \theta \leq 4\pi$  of the equation

$$\sin^2 \theta + 3 \cos \theta = 3 \text{ is}$$

- A 0
- B 1
- C 2
- D 3
- E 4
- F 5
- G 6

3. The perpendicular bisector of the line segment joining the points  $(2, -6)$  and  $(5, 4)$  cuts the  $x$ -axis at the point with  $x$ -coordinate

A  $\frac{1}{20}$

B  $\frac{1}{6}$

C  $\frac{1}{3}$

D  $\frac{19}{5}$

E  $\frac{41}{6}$

4. The complete set of values of  $x$  for which  $(x^2 - 1)(x - 2) > 0$  is

A  $x < -1, 1 < x < 2$

B  $x < -1, x > 2$

C  $-1 < x < 2$

D  $x < 1, x > 2$

E  $-1 < x < 1, x > 2$

5. Given that  $y = -\log_{10}(1 - x)$  for  $x < 1$ , find  $x$  in terms of  $y$ .

A  $x = -\frac{1}{\log_{10}(1-y)}$

B  $x = 1 + \log_{10} y$

C  $x = 1 - \log_{10} y$

D  $x = 1 - 10^{-y}$

E  $x = 10^{-y} - 1$

F  $x = 10^{1-y}$

6. It is given that  $x + 2$  is a factor of  $x^3 + 4cx^2 + x(c + 1)^2 - 6$ .

The sum of the possible values of  $c$  is

A  $-10$

B  $-6$

C  $0$

D  $6$

E  $10$

7. A bag contains  $n$  red balls,  $n$  yellow balls, and  $n$  blue balls.

One ball is selected at random and not replaced.

A second ball is then selected at random and not replaced.

Each ball is equally likely to be chosen.

The probability that the two balls are **not** the same colour is

A  $\frac{n-1}{3n-1}$

B  $\frac{2n-2}{3n-1}$

C  $\frac{2n}{3n-1}$

D  $\frac{(n-1)^3}{27(3n-1)^3}$

E  $\frac{3(n-1)}{3n-1}$

F  $\frac{n^3}{27(3n-1)^3}$

8. Given that  $a^x b^{2x} c^{3x} = 2$ , where  $a$ ,  $b$ , and  $c$  are positive real numbers, then  $x =$

A  $\log_{10} \left( \frac{2}{a+2b+3c} \right)$

B  $\frac{\log_{10} 2}{\log_{10}(a+2b+3c)}$

C  $\frac{2}{\log_{10}(a+2b+3c)}$

D  $\frac{2}{a+2b+3c}$

E  $\log_{10} \left( \frac{2}{ab^2c^3} \right)$

F  $\frac{\log_{10} 2}{\log_{10}(ab^2c^3)}$

G  $\frac{2}{\log_{10}(ab^2c^3)}$

H  $\frac{2}{ab^2c^3}$

9. The roots of the equation  $2x^2 - 11x + c = 0$  differ by 2. The value of  $c$  is

A  $\frac{105}{8}$

B  $\frac{113}{8}$

C  $\frac{117}{8}$

D  $\frac{119}{8}$

10. The curve  $y = \cos x$  is reflected in the line  $y = 1$  and the resulting curve is then translated by  $\frac{\pi}{4}$  units in the positive  $x$ -direction. The equation of this new curve is

A  $y = 2 + \cos\left(x + \frac{\pi}{4}\right)$

B  $y = 2 + \cos\left(x - \frac{\pi}{4}\right)$

C  $y = 2 - \cos\left(x + \frac{\pi}{4}\right)$

D  $y = 2 - \cos\left(x - \frac{\pi}{4}\right)$

11. The sum of the roots of the equation  $2^{2x} - 8 \times 2^x + 15 = 0$  is

A 3

B 8

C  $2 \log_{10} 2$

D  $\log_{10} \left(\frac{15}{4}\right)$

E  $\frac{\log_{10} 15}{\log_{10} 2}$



12. The cross-section of a triangular prism is an equilateral triangle with side  $2x$  cm. The length of the prism is  $d$  cm.

Let the total surface area of the prism be  $T$  cm<sup>2</sup>. Given that the volume of the prism is  $T$  cm<sup>3</sup>, which one of the following is an expression for  $d$  in terms of  $x$ ?

A  $\frac{x}{2x-3}$

B  $\frac{3x}{3x-2\sqrt{3}}$

C  $\frac{2x}{x-4\sqrt{3}}$

D  $\frac{2x}{x-2\sqrt{3}}$

E  $\frac{2x}{x-\sqrt{3}}$

13. How many real roots does the equation  $x^4 - 4x^3 + 4x^2 - 10 = 0$  have?

A 0

B 1

C 2

D 3

E 4

14.  $a, b, x,$  and  $y$  are real and positive.

$a$  and  $b$  are constants.

$x$  and  $y$  are related.

A graph of  $\log y$  against  $\log x$  is drawn.

For which one of the following relationships will this graph be a straight line?

A  $y^b = a^x$

B  $y = ab^x$

C  $y^2 = a + x^b$

D  $y = ax^b$

E  $y^x = a^b$

15. The smallest possible value of  $\int_0^1 (x - a)^2 dx$  as  $a$  varies is

A  $\frac{1}{12}$

B  $\frac{1}{3}$

C  $\frac{1}{2}$

D  $\frac{7}{12}$

E 2

16. Given that  $c$  and  $d$  are non-zero integers, the expression  $\frac{10^{c-2d} \times 20^{2c+d}}{8^c \times 125^{c+d}}$  is an integer if

- A  $c < 0$
- B  $d < 0$
- C  $c < 0$  and  $d < 0$
- D  $c < 0$  and  $d > 0$
- E  $c > 0$  and  $d < 0$
- F  $c > 0$  and  $d > 0$
- G  $d > 0$
- H  $c > 0$

17. For what values of the non-zero real number  $a$  does the quadratic equation  $ax^2 + (a - 2)x = 2$  have real distinct roots?

- A All values of  $a$
- B  $a = -2$
- C  $a > -2$
- D  $a \neq -2$
- E No values of  $a$

18. The angle  $x$  is measured in radians and is such that  $0 \leq x \leq \pi$ .

The total length of any intervals for which  $-1 \leq \tan x \leq 1$  and  $\sin 2x \geq 0.5$  is

- A  $\frac{\pi}{12}$
- B  $\frac{\pi}{6}$
- C  $\frac{\pi}{4}$
- D  $\frac{\pi}{3}$
- E  $\frac{5\pi}{12}$
- F  $\frac{\pi}{2}$
- G  $\frac{5\pi}{6}$

19. A geometric series has first term 4 and common ratio  $r$ , where  $0 < r < 1$ .

The first, second, and fourth terms of this geometric series form three successive terms of an arithmetic series.

The sum to infinity of the geometric series is

- A  $\frac{1}{2}(\sqrt{5} - 1)$
- B  $2(3 - \sqrt{5})$
- C  $2(1 + \sqrt{5})$
- D  $2(3 + \sqrt{5})$

20. The coefficient of  $x^2$  in the expansion of  $(4 - x^2)[(1 + 2x + 3x^2)^6 - (1 + 4x^3)^5]$  is

- A 28
- B 72
- C 78
- D 192
- E 240
- F 310
- G 312

**END OF TEST**

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